
Definition: A **sequence** in a set S is a function from $\mathbb{N} \setminus \{0\}$ to S.

Definition: (**Limit of sequences**)
If, $\forall\ \varepsilon > 0$, $\exists N = N(\varepsilon)$ such that $\forall n > N$, $| x_n - x | \leq \varepsilon$,
then a sequence (x_n) of real numbers **converges** to the real number x.

We write $\lim_{n\to\infty} x_n = x$, and say "$x$ is the limit of the sequence (x_n)".

Definition: If a sequence (x_n) does not converge to some real number, then the sequence (x_n) diverges.

Write the negation of convergence using quantifiers.

Examples

1. Prove that $\lim_{n\to\infty} \frac{1}{n} = 0$.

2. Prove that $\lim_{n\to\infty} 1 = 1$.

3. Prove that $\lim_{n\to\infty} \frac{3}{2n+1} = 0$.

4. Prove that $\lim_{n\to\infty} \frac{2n+1}{n+1} = 2$.

5. Prove that the sequence $a_n = 1 + (-1)^n$ is divergent.